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Published in:
IEEE Transactions on Power Systems

Link to article, DOI:
10.1109/TPWRS.2017.2699687

Publication date:
2018

Document Version
Peer reviewed version

Value of Flexible Resources, Virtual Bidding, and Self-Scheduling in Two-Settlement Electricity Markets With Wind Generation – Part I

Jalal Kazempour, *Member, IEEE*, and Benjamin F. Hobbs, *Fellow, IEEE*

***Abstract*—Part one of this two-part paper presents new models for evaluating flexible resources in two-settlement electricity markets (day-ahead and real-time) with uncertain net loads (demand minus wind). Physical resources include wind together with fast- and slow-start demand response and thermal generators. We also model financial participants (virtual bidders). Wind is stochastic, represented by a set of scenarios. The two-settlement system is modeled as a two-stage process in which the first stage involves unit commitment and tentative scheduling, while the second stage adjusts flexible resources to resolve imbalances. The value of various flexible resources is evaluated through four two-settlement models: i) an equilibrium model in which each player independently schedules its generation or purchases to maximize expected profit; ii) a benchmark (expected system cost minimization); iii) a sequential equilibrium model in which the independent system operator (ISO) first optimizes against a deterministic wind power forecast; and iv) an extended sequential equilibrium model with self-scheduling by profit-maximizing slow-start generators. A tight convexified unit commitment allows for demonstration of certain equivalencies of the four models. We show how virtual bidding enhances market performance, since, together with self-scheduling by slow-start generators, it can help a deterministic day-ahead market to choose the most efficient unit commitment.**

Index Terms– Operational flexibility, wind uncertainty, equilibrium, day-ahead, real-time, demand response, virtual bidding, self-scheduling.

NOTATION

Indices:

d	Index for loads
f	Index for virtual bidders
i	Index for generators
k	Index for demand response blocks
n, m	Indices for nodes
s	Index for wind generation scenarios in real-time market
t	Index for hours

Sets:

$\mathcal{F}$	Set of fast-start dispatchable generators
$\mathcal{FDR}$	Set of fast demand response (DR) providers
$\mathcal{S}$	Set of slow-start dispatchable generators
$\mathcal{SS}$	Set of slow-start dispatchable generators who self-schedule
$\mathcal{SDR}$	Set of slow DR providers
$\mathcal{W}$	Set of wind power generators
Ω_n	Set of nodes connected to node n
Ψ_n	Set of generators/loads/arbitragers located at node n

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Constants:

ϕ_s	Probability of scenario s
$B_{n,m}$	Susceptance of transmission line connected node n to node m [S]
C_i	Production cost of dispatchable generator i [\$/MWh]
C_i^{SU}	Start-up cost of dispatchable generator i [\$]
$C_{d,k,t}^{\downarrow}$	Downward DR provision cost of block k of load d in hour t [\$/MWh]
$C_{d,k,t}^{\uparrow}$	Upward DR provision utility of block k of load d in hour t [\$/MWh]
$D_{d,k,t}^{\downarrow}$	Maximum downward DR of block k of load d in hour t [MW]
$D_{d,k,t}^{\uparrow}$	Maximum upward DR of block k of load d in hour t [MW]
$L_{d,t}$	Demand level of load d in hour t [MW]
$\underline{P}_i$	Minimum power output of generator i [MW]
$\overline{P}_i$	Capacity of dispatchable generator i [MW]
$F_{n,m}$	Transmission capacity of line connected node n to node m [MW]
R_i^{D}	Ramp-down limit of generator i [MW/h]
R_i^{U}	Ramp-up limit of generator i [MW/h]
$W_{i,t}^{\text{DA}}$	Forecast in day-ahead market for production by wind generator $i \in \mathcal{W}$ in hour t [MW]
$W_{i,t,s}^{\text{RT}}$	Forecast in real-time market for production by wind generator $i \in \mathcal{W}$ in hour t under scenario s [MW]

Primal variables (day-ahead market):

$a_{n,m,t}^{\text{DA}}$	Power flow from node n to node m in hour t [MW]
$c_{i,t}^{\text{DA}}$	Start-up cost of dispatchable generator i in hour t [\$]
$d_{d,k,t}^{\text{DA}\downarrow}$	Downward DR of block k of load d in hour t [MW]
$d_{d,k,t}^{\text{DA}\uparrow}$	Upward DR of block k of load d in hour t [MW]
$p_{i,t}^{\text{DA}}$	Power production of generator i in hour t [MW]
$u_{i,t}^{\text{DA}}$	Relaxed commitment status of dispatchable generator i in hour t
$v_{f,t}^{\text{DA}}$	Trading quantity of virtual bidder f in hour t [MW]
$\theta_{n,t}^{\text{DA}}$	Voltage angle of node n in hour t [rad]

Primal variables (real-time market):

$a_{n,m,t,s}^{\text{RT}}$	Power transferred from node n to node m in hour t under scenario s [MW]

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2017.2699687, IEEE Transactions on Power Systems

$c_{i,t,s}^{\mathrm{RT}}$ Start-up cost for fast-start dispatchable generator $i \in \mathcal{F}$ in hour t under scenario s [\$]

$d_{d,k,t,s}^{\mathrm{RT}\downarrow}$ Downward DR adjustment of block k of load d in hour t under scenario s [MW]

$d_{d,k,t,s}^{\mathrm{RT}\uparrow}$ Upward DR adjustment of block k of load d in hour t under scenario s [MW]

$p_{i,t,s}^{\mathrm{RT}}$ Power production adjustment of generator i in hour t under scenario s [MW]

$u_{i,t,s}^{\mathrm{RT}}$ Commitment status adjustment of fast-start dispatchable generator $i \in \mathcal{F}$ in hour t under scenario s

$v_{f,t}^{\mathrm{RT}}$ Trading quantity of virtual bidder f in hour t [MW]

$\theta_{n,t,s}^{\mathrm{RT}}$ Voltage angle of node n in hour t, scenario s [rad]

Dual variables:

$\lambda_{n,t}^{\mathrm{DA}}$ Locational marginal price (LMP) in day-ahead market at node n in hour t [\$/MWh]

$\lambda_{n,t,s}^{\mathrm{RT}}$ Probability-weighted LMP in real-time market at node n in hour t under scenario s [\$/MWh]

μ, ρ Set of dual variables in day-ahead and real-time markets, respectively

I. Introduction

INSTALLED wind capacity in the US approached 75 GW by 2015, of which 53% was added since 2010 [1]. This rapid penetration is due to cost improvements and supporting environmental policies at the federal and state levels. However, uncertainty and variability in wind output pose serious operational challenges in short-term, i.e., day-ahead (DA) and real-time (RT), markets [2]. Specifically, the need to make DA unit commitment (UC) decisions before net loads are known, together with a lack of RT operational flexibility, can lead to cost increases, volatile prices, and even violation of energy balances.

In markets with significant wind power penetration, a major concern is that a *deterministic* clearing of the DA market may result in inefficient schedules and thereby increased expected (probability weighted) system cost (or, more generally, reduced market surplus or "social welfare", which equals the value to consumers minus generation costs), relative to an optimal stochastic schedule. Flexible resources can potentially reduce the cost consequences of suboptimal DA schedules. We consider three types of physical flexible resources in this paper, including fast-start dispatchable generators (peaking units), slow demand response (DR) providers (requiring DA schedules), and fast DR providers (which are dispatchable in RT). We also consider financial players in this paper. These players take the form of *virtual bidders* or arbitragers who own no physical assets and buy (sell) in the DA market and then sell (buy) the same amount back in RT [3]-[6], as illustrated in Fig. 1. In US electricity markets, e.g., CAISO and PJM, they are allowed to trade through intertemporal energy arbitrage between DA and RT. The bidding action of these arbitragers is sometimes called "convergence bidding" (as in CAISO), as these players usually help the DA and RT market prices to converge in expectation [7]. Virtual bidding (VB) can occur by buying (or selling) a fixed MWh quantity in a particular interval of the day-ahead market at whatever market price

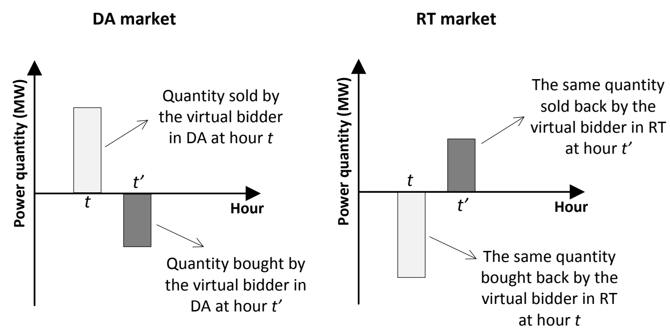

Fig. 1. Virtual bidding (VB): The virtual bidder buys (sells) in the DA market and then sells (buys) the same amount back in RT.

occurs, and then sell (buy) the same quantity back in the corresponding intervals in RT. Or, more commonly, the virtual bidder can submit a bid to up to a stated amount of MWh in the DA market at a maximum given price (or offer to sell an amount at a minimum price); then in the real-time market, the virtual bidder zeroes out its position by selling (buying) back the quantity of MWh that it bought (sold) DA. In general, virtual bidders can enhance the markets ability to cope with forecast errors by increasing liquidity and giving a means for players to provide information to the market. Statistically significant improvements in market efficiency have been found in [7] and [8]. A PJM report in 2015 ([9], discussed in [10]) reviews the operation of virtual bidders in that energy market, discusses their roles, and offers some recommendations.

In this paper, we are interested in answering two main technical questions. First, how do we evaluate the economic benefits of adding various types of flexible resources to manage forecast errors? Secondly, can physical flexible resources and virtual bidders improve deterministic DA schedules? More specifically, can VB alone make the deterministic DA schedules fully efficient, i.e., identical to those obtained from an ideal *stochastic* DA optimization model? If not, can the DA schedules become fully efficient with VB combined with slow-start dispatchable generators who self-schedule? These power economics questions motivate the following methodological question: how can DA-RT markets with VB and self-scheduling be simulated? To answer all of these questions, a *two-settlement* setup including DA and RT markets is considered. Then, we propose four different two-settlement market-clearing equilibrium models that represent different philosophies of market design (deterministic vs. stochastic day-ahead; central market clearing vs. self-scheduling), and subsequently explore their relationships.

It is common in the literature to simulate markets through *optimization* models, taking advantage of the well-known result that competitive markets without market failures can be simulated by maximizing (expected) market surplus [11]. One of our four models uses this approach to define an efficient baseline for comparison with our other models. Optimization models have often been used to explore the DA-RT price convergence and other properties of two-settlement electricity markets under uncertainty (see the survey in [12]). For instance, a two-settlement electricity market-clearing mechanism is proposed in [13] based on stochastic programming, and is proven to be revenue adequate for the independent system operator (ISO) in each possible RT scenario. Reference [12] extends that model, using penalties in the DA model to produce DA prices and quantities equaling expected RT values. Unlike our market models, neither considers the possible arbitraging role of virtual bidders, nor do they consider unit commitment decisions.

In contrast to the optimization approach, we instead directly formulate most of our market problems as *equilibrium* models. One reason is that we want to model the possibility of market failures, in which market surplus is not maximized; we also wish to incorporate VB which might contribute to the correction of market failures. Our two-settlement equilibrium models consider trading by virtual bidders between the DA and RT markets together with the ISO who clears those two markets separately. Several papers in the literature use equilibrium models to simulate two-settlement markets in power and other sectors, but for different purposes than ours. For instance, [14] proposes a stochastic two-settlement equilibrium model to analyze the effects of policies that alter power market incentives such as renewable subsidies. Market power was also the focus of [15], which shows that competitiveness of markets is improved by including one or more futures market; they assume that virtual bidders ensure that DA and expected prices are equal. Building on that work, [16] proposes a two-settlement electricity market considering not only market power but also flow congestion, demand uncertainty, and system contingencies, and uses it to quantify the distorting impacts of DA zonal pricing.

In the past, research on wind integration costs emphasized the increased need for operating reserves [17]-[20] and transmission [21]; changes in system costs as wind penetration increases [22]-[25]; the quantification of the value of physical flexible resources in wind-integrated markets [26]-[29]. In contrast to our work, these papers do not represent two-settlement markets that are financially arbitraged; as we will show later, including VB can dramatically change solutions and costs.

There is also work that addresses the impact of VB on two-settlement markets with stochastic generation. Empirical work in California indicates that VB has resulted in price convergence between DA and RT, and improved efficiency [7], [8]. Meanwhile, reference [30] describes the circumstances under which VB cannot improve market efficiency, and provides a specific case study in California to illustrate these circumstances. Equilibrium-based models of markets have been used to explore possible reasons for that outcome. For instance, reference [31] points out different inefficiencies arising from the German balancing market designs, and describes arbitrage opportunities between the spot market and the balancing mechanism. Meanwhile, reference [32] considers a dispatch model without UC constraints and investigates how VB reduces merit-order inefficiency in a forward market. In particular, they show that VB can potentially improve the conventional two-stage market, except in those cases where the classical merit-order dispatch needs to be violated. In the classical merit-order dispatch, the ISO ranks the generators from the cheapest to most expensive (in terms of variable cost), and then dispatches them in that order. In practice, departures from merit-order dispatch are required to meet other constraints at least-cost. Examples of such constraints include transmission capacity, ramping limits, and requirements for operating reserves. Our result can be viewed as a generalization of [32] in which we consider (i) UC constraints and costs as well as transmission, which result in out-of-merit dispatch, as well as (ii) incentives for and impacts of self-scheduling.

Considering the context above, the main contributions of this two-part paper are as follows. We propose four distinct two-settlement market-clearing models, including three equilibrium models and one optimization problem. Within these models, UC constraints are enforced through a tight convex relaxation; its convexity allows for demonstration of certain properties and equivalencies among the models that would not be possible with discrete (binary) commitment models. These four models allow the value of various flexible resources to be quantified under different market assumptions. Another motivation of this study is to ask whether the ISO and/or all generators need to do stochastic unit commitment to achieve the optimal solution (least expected system cost), or whether virtual bidders in combination with stochastically-optimized self-scheduling by a few slow-start resources alone are enough to obtain it. This question is addressed by simulating VB and self-scheduling of slow-start resources and their impacts on DA schedules and costs within the equilibrium models.

The paper is organized as follows. In section II, we describe the various market players as well as our modeling features and assumptions. In section III, we compare the structure and motivation of the four proposed two-settlement market models. Section IV provides the mathematical formulations of the first of the four models, which is the two-settlement equilibrium model. Section IV concludes the paper. In the second part of this two-part paper, we present the other three models, and then apply all four models to a small test system and to the 24-node IEEE Reliability Test System to quantify the value of flexible resources and the impact of VB and self-scheduling on the efficiency of DA schedules.

II. Model Assumptions and General Structure

The following market players are considered in constructing our equilibrium models of two-settlement markets: slow-start and fast-start dispatchable generators; wind power facilities with variable generation; slow and fast DR providers; arbitragers (virtual bidders); the grid operator; and consumers (loads). A slow-start dispatchable (thermal) generator makes its commitment decisions in the DA market, while those decisions are fixed in the RT market. In other words, it is only allowed to alter its RT production level while constrained by its fixed commitment and ramping limits. However, unlike slow-start generators, a fast-start dispatchable generator can change its commitment decisions in RT as well as its output.[1] A wind power generator is a unit with zero marginal cost, and usually has the ability to be curtailed. Renewable incentives are not considered in this paper.

[1] It is possible to divide RT markets into separate real-time unit commitment and dispatch-only RT markets, for example, as in the California market, but this refinement is left for future research.

As for other market players, we consider two types of DR providers: slow and fast [33]-[35]. Slow DR providers, e.g., an industrial plant, can only change their consumption schedule in the DA market. However, fast DR providers, e.g., DR aggregators with HVAC controls, can be scheduled DA and then redispatched at short notice for RT balancing purposes. It is assumed that all DR providers can provide both upward and downward DR services, i.e., they can either increase and/or decrease their consumption levels to some extent.

Meanwhile, the ISO is, in essence, a spatial arbitrager or trader who moves power around to maximize the value of the transactions, while respecting the physical constraints of the grid [36]. The final market party is consumers; for simplicity, we assume all system loads to be deterministic and perfectly inelastic with respect to price, so the only forecast errors occur for wind power generators. A more general formulation to include load uncertainty is a straightforward extension [36].

We now review some general assumptions about the market players' information, behavior, and costs.

First, we assume that if the ISO solves a deterministic model to clear the DA market, it uses a single wind forecast (actually, a single set of offers of MW production by wind plants) and treats it as if it is a perfect prediction of actual wind availability.[2]

Second, we make the simplifying assumption that in the RT markets, the real-time conditions for the entire day, such as hourly wind production, become known at the beginning of the day, but after DA commitments are made. That is, the actual wind realization for the full 24 hours becomes known at midnight, and is selected from one of the set of possible scenarios. More complex assumptions (such as Markov decision process in real time [37]) are possible and would result in more hedging in RT commitment and dispatch decisions [38], but at the cost of a much more complex model. Our simplifying assumption has been made by many other wind integration models (e.g., [39]), and does not alter the fundamental fact that DA scheduling is done under much more forecast uncertainty than real-time operations. Wind scenario generation and/or reduction techniques are outside the scope of this paper.

Third, we assume that all market players act competitively. That is, when bidding in an ISO-operated market, they offer truthfully at quantities equal to their available capacities at prices identical to their marginal costs. On the other hand, if they self-schedule, we assume that they do so non-strategically, by assuming that their decisions will not affect prices. In addition, all players are assumed to be expected-profit maximizers. Generalizations that allow for risk aversion (e.g., [14]) and Nash-Cournot market power (e.g., [16]) are straightforward. However, if the market requires submission of a single non-decreasing DA offer curve for each unit that is to apply to all hours, Cournot price-quantity equilibria might be inconsistent with this requirement, and more complex supply function equilibria models could be, in theory, more appropriate [40].

Fourth, the generation-side constraints in this work are represented by a tight relaxed version of the UC problem [41], as the resulting model is convex and therefore avoids equilibrium existence problems that arise with UC models with 0/1 binary variables. This particular relaxed model provides more accurate UC cost estimates than either using pure dispatch models (that ignore 0/1 commitment decisions) or simply relaxing the 0/1 binary restriction in standard UC formulations. In particular, the feasible region of the UC problem is tightened by including the relaxed commitment variables within ramping constraints as shown below. We include startup costs, Pmin restrictions and costs, and ramp limits, but other UC constraints, such as minimum on- and off-times [42], are omitted for simplicity. Intuitively, this relaxation allows a continuous portion of a plant (or a set of identical plants) to be committed at any given time, such as 570 MW out of a 1000 MW unit. The minimum and maximum outputs of the generator, as well as its ramp rate limits, are proportional to the committed capacity. Startup costs are incurred when the quantity of committed capacity increases from interval to interval, and are proportional to the amount of increase.[3] We also exclude operating reserve products from our problem since we do not address forced outages of the physical assets here. However, a generalization including reserves is easily derived, but would complicate our analysis without significantly changing its conclusions.

III. The Four Two-Settlement Market Models

The following four DA-RT market-clearing models are considered in this paper:

- *Stoch-MP*: Multi-player stochastic two-settlement equilibrium model. This is a theoretical model, not implemented in any market, in which the ISO supervises a market-clearing process both DA and in RT between parties who each determine on their own what quantities they want to buy or sell ("self-schedule") in each of the markets in response to the DA and RT prices. It is assumed that the ISO adjusts market prices and the parties adjust their self-schedules to result in an equilibrium situation in which no party can increase its expected (probability-weighted) profit by adjusting the amounts it sells in any market.

[2]This simplifies the reality of ISO DA market scheduling processes today. These are actually the result of a complex interaction of deterministic MW offers by wind generators in the DA financial markets, which result in schedules of thermal generators that are subject to alteration by so-called "residual unit commitment" (RUC) processes that instead are based on an ISO wind forecast. RUC processes complement the financial market-clearing process, and are intended to assure operators that enough physical resources have been committed day ahead to meet plausible physical load and renewable contingencies, irrespective of the load and resources (including offered wind) that cleared in the financial market. The roles of deterministic wind offers and ISO forecasts become even more complicated if some renewable resources are scheduled in the financial market based on ISO forecasts (e.g., the CAISO's "Participating Intermittent Resource Program").

[3]Our numerical tests of this approximation [41] show that it is most accurate when there are multiple generating units of a given type, so that the error resulting from (for example) assuming that, say, 57% of capacity is on line rather than forcing the solution to be either exactly, say, 5 or 6 out of 10 identical units is negligible. Thus, the approximation is better for larger systems with many similar units than for smaller systems whose capacity is dominated by a handful of large generators. Those tests also indicate that the approximation yields much better estimates of total production costs, plant capacity factors, and prices than does, for example, load duration curve-type models that disregard unit commitment constraints entirely.

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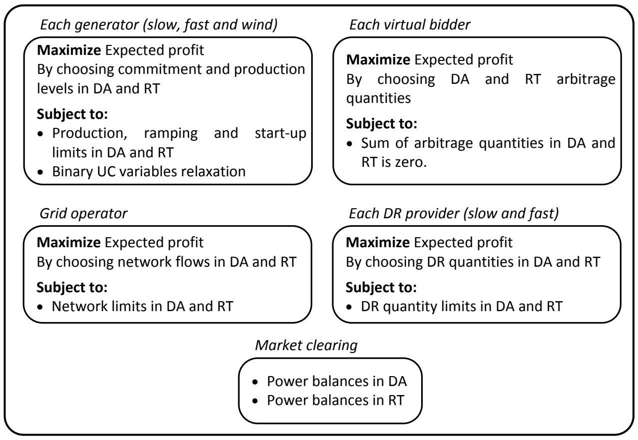

Fig. 2. The structure of multi-player stochastic two-settlement equilibrium model ($Stoch\text{-}MP$)

- $Stoch\text{-}Opt$: Stochastic optimization model, which minimizes the total expected system cost (baseline). This is also a theoretical model that has not been implemented in actual markets, in which a central operator solves a stochastic unit commitment problem day ahead to determine an optimal day-ahead energy schedule, followed by real-time resolution of imbalances by the ISO. There are no self-schedules.
- Seq: Sequential two-settlement equilibrium model. This market model as well as the next one are simplified representations of how US markets now operate, in which a market operator schedules thermal generation in the DA market considering only a single (deterministic) demand and wind forecast, and virtual bidders are allowed to buy in the DA market and sell back in the real-time market. This is a simplification of how US markets actually work, in which the ISO's demand and wind forecasts are used in DA "residual unit commitment" markets to ensure that enough capacity is committed, while demand bids and wind offers that can depend on price are considered in the financial DA market.
- $Seq\text{-}SS$: Extended Seq with self-scheduling slow-start generators. Like the previous model, this is a simplified representation of today's markets in the US. This model also considers the ability of generators in these markets to self-schedule in the DA market and then adjust output in the RT market if they find that more profitable than having the ISO schedule its output.

Note that $Stoch\text{-}MP$, Seq and $Seq\text{-}SS$ are equilibrium models, while $Stoch\text{-}Opt$ is an optimization problem, representing the situation in which the ISO schedules all generation and DR in both DA and RT using stochastic programming. Across these four models, we consider identical wind forecasts and error distributions. In each model, actual RT wind realizations are assumed to follow a distribution defined by a set of scenarios and probabilities; what market parties assume about the wind distribution in the DA market depends on the model, as explained below. However, we assume *perfect* VB in all four models, i.e., the virtual bidders in DA have perfect knowledge of the distribution of RT prices, and eliminate any arbitrage profits. That is, they will buy or sell in the two markets until the DA prices equal the expected RT prices. The market models differ concerning the role of the ISO as well as which parties consider the RT distribution of prices when determining DA schedules, as explained next.

In $Stoch\text{-}MP$, the ISO only allocates scarce transmission capacity, and does not schedule generation, unlike $Stoch\text{-}Opt$, Seq and $Seq\text{-}SS$. Each market player (generators, DR, and virtual bidders) maximizes its expected profit across the DA and RT markets considering them *simultaneously*, each solving a stochastic profit maximization problem. For instance, a slow-start thermal generator decides DA how much capacity to commit, how much power to sell in the DA and each of RT market (one per wind realization), and how much to generate in order to fulfill those supply schedules. Fast-start thermal generators, in contrast, do not need to make commitment decisions until real time, but can also sell in either DA and/or the RT markets. Wind power generators decide how much to sell DA and in each of the RT markets, given its distribution of potential wind production. Slow DR makes its commitment decisions DA and sells in the DA market, while fast DR waits until real time, but buys/sells power either DA or in RT, whichever is most profitable, thus being able to arbitrage the two markets.[4] The system operator performs a spatial arbitrage function, being able to buy at one node and sell at another at the same time while satisfying transmission constraints. Finally, the temporal arbitrage function is assigned to virtual bidders. Simultaneously solving these profit maximization problems for all players while imposing market clearing forms a stochastic equilibrium problem. The structure of this equilibrium model is schematically depicted in Fig. 2. In this model, it is assumed that each player has the same (and correct) beliefs concerning the distribution of wind generation scenarios and the RT prices that result in each. It can be intuitively expected that the VB has no impact in $Stoch\text{-}MP$ since not only virtual bidders, but also *all* other players have the same knowledge in DA concerning wind scenarios in RT, and can decide to schedule their output or consumption in either market, thus acting as arbitragers themselves.

In $Stoch\text{-}Opt$, a single optimization problem is solved, in which the grid operator uses stochastic programming to co-optimize all generation and DR in both DA and RT markets to minimize the total expected system cost, and in addition calculates DA and RT locational marginal prices (LMPs). The structure of this model is schematically illustrated in Fig. 3. Similar to $Stoch\text{-}MP$, DA and RT outcomes are determined *simultaneously*, so the separate RT scheduling and pricing problems are solved all at once. Independent virtual bidders arbitrage the DA and RT markets in $Stoch\text{-}Opt$, so that the DA and the expected RT prices are the same; however, by performing stochastic optimization for DA and RT simultaneously, the ISO in a sense already performs the arbitrage function, so the presence or absence of virtual bidders will not affect the solution.

By design, $Stoch\text{-}Opt$ obtains the fully efficient DA sched-

[4]An alternative formulation could allow the slow DR to also sell in the RT market, but this would not change the basic market results.

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Minimize [system cost in DA] + [expected system cost in RT]

Subject to:
- Production, ramping and start-up limits of generators in DA and RT
- Generators' binary UC variables relaxation
- Sum of virtual arbitrage quantities in DA and RT is zero
- Network limits in DA and RT
- DR quantity limits in DA and RT
- Power balances in DA and RT

Fig. 3. The structure of stochastic optimization model, which minimizes the total expected system cost ($Stoch\text{-}Opt$)

ules and the least-possible expected system cost. We demonstrate later that under mild convexity conditions, the solutions of $Stoch\text{-}MP$ and $Stoch\text{-}Opt$ are the same, in the sense that any solution of one model is also a solution to the other. This implies that if all market parties independently optimize their schedules, costs are convex, no one possesses market power, and everyone has the same expectations about the distribution of RT prices, then the stochastic equilibrium among market parties who self-schedule against DA and RT prices results in the minimum expected cost of meeting demand.

In Seq, unlike $Stoch\text{-}MP$ and $Stoch\text{-}Opt$, a *sequential* market clearing is simulated in which the ISO first clears the DA market deterministically considering just the single deterministic wind forecast. In the basic version of this model, the operator schedules all generation and DR. Then, in real-time, the ISO clears the RT market for each wind generation scenario by scheduling deviations for any generators and DR who are not self-scheduled. In the basic version of this model, the virtual bidders are assumed to be the *only* players that have perfect knowledge in DA concerning the full distribution of RT prices resulting from the wind generation scenarios. Therefore, each virtual bidder considers both DA and RT markets simultaneously and maximizes its expected profit; in equilibrium, this arbitrage will eliminate any difference between DA prices and expected RT prices. In contrast, other players such as generators are naive in the sense that they allow the ISO to dispatch them in the DA based on the deterministic wind forecast in that market, and then are recommitted and/or redispatched by the ISO in the RT market based on actually realized wind scenario in that market. Fig. 4 depicts the structure of this model. Unlike $Stoch\text{-}MP$ and $Stoch\text{-}Opt$, it can be intuitively expected that the VB potentially impacts the market-clearing outcomes in Seq. Accordingly, we solve Seq with and without virtual bidders, and then compare the results obtained, i.e., DA schedules and total system cost, with those obtained from $Stoch\text{-}MP$ and $Stoch\text{-}Opt$. As we will show, without VB, the sequential solutions of Seq are likely to be inefficient compared to the ideal (cost-minimizing) solutions of $Stoch\text{-}MP$ and $Stoch\text{-}Opt$, but VB can improve the performance of Seq. In some (but not all) cases, VB eliminates all the inefficiencies and yields the same DA commitments as the stochastic solution, even though the ISO is solving a deterministic market model in the DA market.

In $Seq\text{-}SS$, we consider an extension of Seq in which selected slow-start generators are allowed to self-schedule [43], under the assumption that they (like virtual bidders)

DA market clearing

Minimize System cost in DA
By choosing commitment and production levels of generators, DR schedules and network flows in DA

Subject to:
- Production, ramping and start-up limits of generators in DA
- Generators' binary UC variables relaxation
- Network limits in DA
- DR quantity limits in DA
- Power balances in DA

RT market clearing for each wind scenario

Minimize Probability-weighted system cost in RT
By choosing production levels of generators, commitment of fast-start generators, fast DR schedules and network flows in RT

Subject to:
- Production, ramping and start-up limits of generators in RT
- Fast-start generators' binary UC variables relaxation
- Network limits in RT
- Fast DR quantity limits in RT
- Power balances in RT

Each virtual bidder

Maximize Expected profit
By choosing DA and RT arbitrage quantities

Subject to:
- Sum of arbitrage quantities in DA and RT is zero.

Fig. 4. The structure of sequential two-settlement equilibrium model (Seq)

DA market clearing

Minimize System cost in DA
By choosing commitment and production levels of generators (excluding self-scheduling slow-start generators), DR schedules and network flows in DA

Subject to:
- Production, ramping and start-up limits of generators in DA
- Generators' binary UC variables relaxation
- Network limits in DA
- DR quantity limits in DA
- Power balances in DA

RT market clearing for each wind scenario

Minimize Probability-weighted system cost in RT
By choosing production levels of generators, commitment of fast-start generators, fast DR schedules and network flows in RT

Subject to:
- Production, ramping and start-up limits of generators in RT
- Fast-start generators' binary UC variables relaxation
- Network limits in RT
- Fast DR quantity limits in RT
- Power balances in RT

Each virtual bidder

Maximize Expected profit
By choosing DA and RT arbitrage quantities

Subject to:
- Sum of arbitrage quantities in DA and RT is zero.

Each self-scheduling slow-start generator

Maximize Expected profit
By choosing commitment and production levels in DA and RT

Subject to:
- Production, ramping and start-up limits in DA and RT
- Binary UC variables relaxation

Fig. 5. The structure of extended Seq with self-scheduling slow-start generators ($Seq\text{-}SS$)

correctly anticipate the RT distribution of prices. They optimize their schedules in the DA and RT markets based on the DA and RT prices calculated by the ISO. Thus, this model includes in a single equilibrium model: (i) deterministic scheduling of all DR and some generators by the operator, as in Seq; (ii) independent self-scheduling by selected slow-start

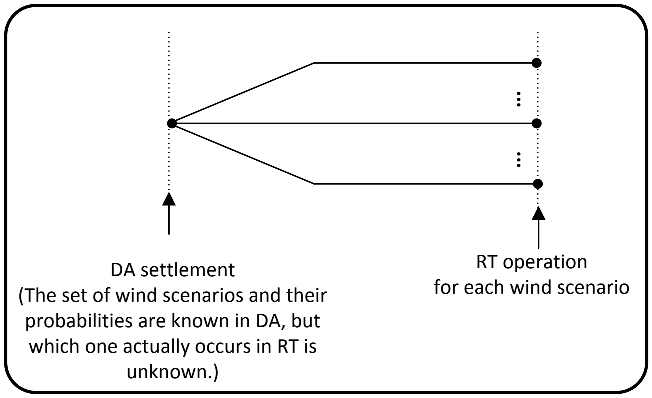

Fig. 6. The decision sequence in stochastic models (i.e., $Stoch\text{-}MP$ and $Stoch\text{-}Opt$)

generators (the so-called self-scheduling generators), based on expected profit-maximizing, similar to $Stoch\text{-}MP$; and (iii) virtual bidders as in $Stoch\text{-}MP$, $Stoch\text{-}Opt$ and Seq. The structure of this model is illustrated in Fig. 5. The results of $Seq\text{-}SS$ (in the second of this two-part paper) show that this extension tends to improve the solutions of Seq further, if they are not already efficient. This model enables us to address whether a deterministic DA market followed by RT markets with VB and self-scheduling can result in fully efficient DA schedules and least-possible system cost, as in $Stoch\text{-}MP$ and $Stoch\text{-}Opt$.

Among these four models, $Stoch\text{-}Opt$ has the attractive theoretical property of achieving the lowest expected operating cost, under the assumption that the ISO can obtain the stochastic information (e.g., probability distributions) required for *stochastic clearing*. However, this assumption is incompatible with the current practice of real-world electricity markets, and its implementation would place a large burden on the ISO to develop this information and to obtain stakeholder consent for the procedures involved. In contrast, and in line with the structure of current markets, the ISO sticks to *deterministic clearing* in Seq and $Seq\text{-}SS$, while instead allowing the virtual bidders and self-scheduling generators to adjust energy schedules to deal with uncertainty.[5]

We now describe at what point of time each model is solved. The decision sequence in stochastic models, i.e., $Stoch\text{-}MP$ and $Stoch\text{-}Opt$, is illustrated in Fig. 6. In these two ideal models, the set of RT wind scenarios and their probabilities are known at the DA stage. Therefore, the DA schedule as well as the RT schedules by scenario are solved in one shot, so that DA decisions are made fully recognizing what adaptations might happen in real time. Meanwhile, the decision sequence in sequential models, i.e., Seq and $Seq\text{-}SS$, is depicted in Fig. 7. Similar to the stochastic models, Seq and $Seq\text{-}SS$ are solved day ahead at once. The reason for this is that although the ISO clears DA and RT markets sequentially (the upper box in Fig. 7), the virtual bidders (and self-scheduling generators in $Seq\text{-}SS$) make their DA and RT decisions simultaneously, recognizing their interdependency and the uncertainty in RT (the lower box in Fig. 7). (This is because a decision to sell X MW in the DA market irrevocably binds the virtual bidder to buying back the same amount in RT.) When calculating the overall market outcomes in these two models, however, we consider the decision-making problems of the ISO, virtual bidders and self-scheduling generators together, forming an equilibrium model. This allows us to assess the value of informed VB and self-scheduling. This equilibrium calculation simulates the receipt by the ISO of the offers and bids of virtual bidders and self-scheduled generators, and then the sequential clearing of the DA and RT markets at different points of time.

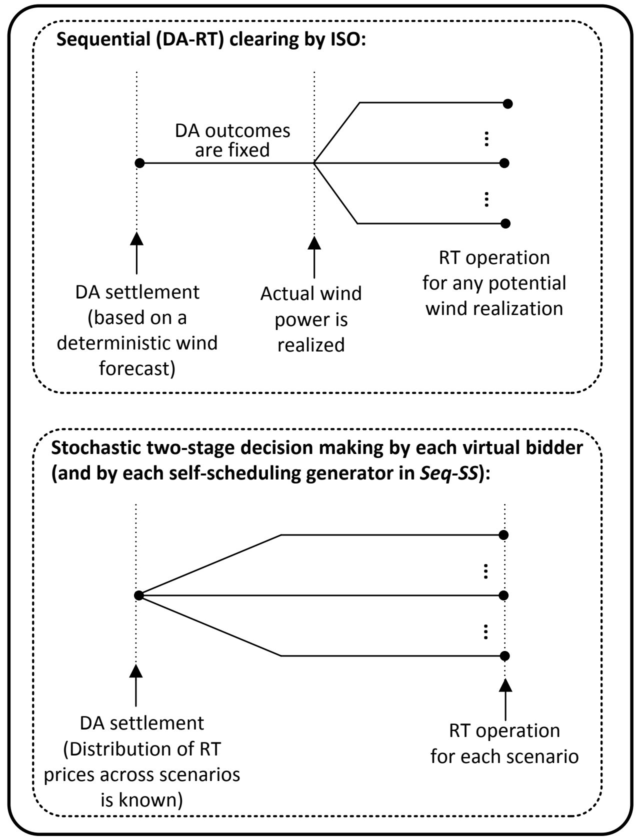

Fig. 7. The decision sequence in sequential models (i.e., Seq and $Seq\text{-}SS$)

It is worth mentioning that we consider the same distribution of uncertain wind production in all four models since each of them are solved at once at the DA stage, assuming the same closing time and thus wind information availability at that stage. All models assume that there is no uncertainty in real time.

The mathematical formulation of $Stoch\text{-}MP$ is provided next. The other three models are described in the companion

[5]What our analysis indicates is that it is possible that a subset of market parties acting based on high quality stochastic information, can achieve the same efficiencies as central stochastic clearing by the ISO. Therefore, despite the theoretical appeal of the latter approach, its practical difficulties together with the efficiency of $Seq\text{-}SS$ suggest that ISOs should not embrace the central stochastic model, but should instead carefully evaluate whether self-scheduling and VB in fact already allow markets to realize most of the potential efficiencies of stochastic scheduling. Although there has been previous works comparing deterministic and stochastic clearing mechanisms [44]-[47], they have not modeled VB and self-scheduling; further theoretical and empirical work is needed before a definite conclusion can be made about whether stochastic information is best incorporated through central ($Stoch\text{-}MP$) or distributed ($Seq\text{-}SS$) types of processes.

paper [48], based on the assumptions described above along with several of the individual market party problems given below.

A. *Stoch-MP: Multi-Player Stochastic Two-Settlement Equilibrium Model*

As illustrated in Fig. 2, several profit-maximization problems are considered in *Stoch-MP*, one per player. These problems are defined in this subsection by (1). Note that dual variables are listed alongside each constraint. First, the profit-maximization problem for each slow-start dispatchable generator $i \in \mathcal{S}$ is given by (1a) below.

$$\left\{ \begin{array}{l} \underset{u^{\text{DA}}_{i,t}, c^{\text{DA}}_{i,t}, p^{\text{DA}}_{i,t}, p^{\text{RT}}_{i,t,s}}{\text{Maximize}} \\ \sum_t \left[p^{\text{DA}}_{i,t} \left(\lambda^{\text{DA}}_{(n:i\in\Psi_n),t} - C_i \right) - c^{\text{DA}}_{i,t} \right] \\ + \sum_{t,s} \phi_s \left[p^{\text{RT}}_{i,t,s} \left(\frac{\lambda^{\text{RT}}_{(n:i\in\Psi_n),t,s}}{\phi_s} - C_i \right) \right] \qquad \text{(1aa)} \\ \text{subject to:} \\ u^{\text{DA}}_{i,t} \, \underline{P}_i \le p^{\text{DA}}_{i,t} \le u^{\text{DA}}_{i,t} \, \overline{P}_i \ : \underline{\mu}_{i,t}, \overline{\mu}_{i,t} \ \forall t \qquad \text{(1ab)} \\ u^{\text{DA}}_{i,t} \, \underline{P}_i \le \left(p^{\text{DA}}_{i,t} + p^{\text{RT}}_{i,t,s} \right) \le u^{\text{DA}}_{i,t} \, \overline{P}_i \ : \underline{\rho}_{i,t,s}, \overline{\rho}_{i,t,s} \\ \qquad \forall t, s \quad \text{(1ac)} \\ -u^{\text{DA}}_{i,(t-1)} \, R^{\text{D}}_i \le \left(p^{\text{DA}}_{i,t} - p^{\text{DA}}_{i,(t-1)} \right) \le u^{\text{DA}}_i \, R^{\text{U}}_i : \mu^{\text{D}}_{i,t}, \mu^{\text{U}}_{i,t} \\ \qquad \forall t \quad \text{(1ad)} \\ -u^{\text{DA}}_{i,(t-1)} \, R^{\text{D}}_i \le \left(p^{\text{DA}}_{i,t} + p^{\text{RT}}_{i,t,s} - p^{\text{DA}}_{i,(t-1)} - p^{\text{RT}}_{i,(t-1),s} \right) \\ \qquad \le u^{\text{DA}}_{i,t} \, R^{\text{U}}_i \quad : \rho^{\text{D}}_{i,t,s}, \rho^{\text{U}}_{i,t,s} \quad \forall t, s \qquad \text{(1ae)} \\ c^{\text{DA}}_{i,t} \ge C^{\text{SU}}_i \left(u^{\text{DA}}_{i,t} - u^{\text{DA}}_{i,(t-1)} \right) \quad : \mu^{\text{SU}}_{i,t} \quad \forall t \qquad \text{(1af)} \\ u^{\text{DA}}_{i,t} \le 1 \quad : \mu^{\text{rlx}}_{i,t} \quad \forall t \qquad \text{(1ag)} \\ u^{\text{DA}}_{i,t} \ge 0; \ c^{\text{DA}}_{i,t} \ge 0 \quad \forall t \end{array} \right\} \ \forall i \in \mathcal{S}. \qquad \text{(1ah)}$$

Objective function (1aa) maximizes the expected profit of slow-start generator $i \in \mathcal{S}$ in the DA and RT markets. Note that $\frac{\lambda^{\text{RT}}_{n,t,s}}{\phi_s}$ refers to the probability-adjusted real-time LMP at node n, hour t under scenario s. Constraints (1ab) and (1ac) bound the production levels in DA and RT markets, respectively. Constraints (1ad) and (1ae) enforce the ramp rate limits of slow-start generator in DA and RT markets, respectively. Finally, constraints (1af)-(1ah) calculate the start-up cost of generator $i \in \mathcal{S}$ and relax its corresponding commitment variables in the DA market.

According to (1a), a slow-start generator makes all commitment decisions in the DA market, with the chosen commitment schedule being imposed in all RT scenarios. It is assumed that DA and RT schedules must both be feasible relative to generation capacity and ramping constraints. However, an alternative formulation can omit the DA constraints (1ab) and (1ad), in which case the DA schedule is purely financial, and the generator can act as an unrestrained financial arbitrager between the DA and RT markets. In the presence of separate virtual players, as in this model, this omission would not affect the equilibrium prices, unit commitment, or total (DA plus RT) dispatch of generators or DR.

The profit-maximization problem for each fast-start dispatchable generator $i \in \mathcal{F}$ is given by (1b) below:

$$\left\{ \begin{array}{l} \underset{u^{\text{DA}}_{i,t}, c^{\text{DA}}_{i,t}, p^{\text{DA}}_{i,t}, u^{\text{RT}}_{i,t,s}, c^{\text{RT}}_{i,t,s}, p^{\text{RT}}_{i,t,s}}{\text{Maximize}} \\ \sum_t \left[p^{\text{DA}}_{i,t} \left(\lambda^{\text{DA}}_{(n:i\in\Psi_n),t} - C_i \right) - c^{\text{DA}}_{i,t} \right] \\ + \sum_{t,s} \phi_s \left[p^{\text{RT}}_{i,t,s} \left(\frac{\lambda^{\text{RT}}_{(n:i\in\Psi_n),t,s}}{\phi_s} - C_i \right) - c^{\text{RT}}_{i,t,s} \right] \qquad \text{(1ba)} \\ \text{subject to:} \\ \text{(1ab), (1ad), (1af)} - \text{(1ah)} \qquad \text{(1bb)} \\ \left(u^{\text{DA}}_{i,t} + u^{\text{RT}}_{i,t,s} \right) \underline{P}_i \le \left(p^{\text{DA}}_{i,t} + p^{\text{RT}}_{i,t,s} \right) \\ \qquad \le \left(u^{\text{DA}}_{i,t} + u^{\text{RT}}_{i,t,s} \right) \overline{P}_i : \underline{\rho}_{i,t,s}, \overline{\rho}_{i,t,s} \quad \forall t, s \qquad \text{(1bc)} \\ -\left(u^{\text{DA}}_{i,(t-1)} + u^{\text{RT}}_{i,(t-1),s} \right) R^{\text{D}}_i \le \\ \quad \left(p^{\text{DA}}_{i,t} + p^{\text{RT}}_{i,t,s} - p^{\text{DA}}_{i,(t-1)} - p^{\text{RT}}_{i,(t-1),s} \right) \le \\ \quad \left(u^{\text{DA}}_{i,t} + u^{\text{RT}}_{i,t,s} \right) R^{\text{U}}_i \quad : \rho^{\text{D}}_{i,t,s}, \rho^{\text{U}}_{i,t,s} \quad \forall t, s \qquad \text{(1bd)} \\ c^{\text{DA}}_{i,t} + c^{\text{RT}}_{i,t,s} \ge C^{\text{SU}}_i \left(u^{\text{DA}}_{i,t} + u^{\text{RT}}_{i,t,s} - u^{\text{DA}}_{i,(t-1)} - u^{\text{RT}}_{i,(t-1),s} \right) \\ \qquad : \rho^{\text{SU}}_{i,t,s} \quad \forall t, s \qquad \text{(1be)} \\ c^{\text{DA}}_{i,t} + c^{\text{RT}}_{i,t,s} \ge 0 \quad : \rho^{\text{c}}_{i,t,s} \quad \forall t, s \qquad \text{(1bf)} \\ u^{\text{DA}}_{i,t} + u^{\text{RT}}_{i,t,s} \le 1 \quad : \rho^{\text{rlx}}_{i,t,s} \quad \forall t, s \qquad \text{(1bg)} \\ u^{\text{RT}}_{i,t,s} \ge 0 \quad \forall t, s \end{array} \right\} \ \forall i \in \mathcal{F}. \qquad \text{(1bh)}$$

Unlike the slow-start generators, the commitment status of each fast-start generator can be adjusted in the RT market. Objective function (1ba) maximizes the expected profit of fast-start generator $i \in \mathcal{F}$ in DA and RT markets. The generation constraints of fast-start generator in the DA market are enforced by (1bb), while (1bc)-(1bh) represent the analogous constraints in the RT market. Note that $u^{\text{RT}}_{i,t,s}$ in (1bg) is appropriately viewed as the incremental commitment in RT relative to the DA commitment.

The profit-maximization problem for each wind power generator $i \in \mathcal{W}$ is given by (1c) below:

$$\left\{ \begin{array}{l} \underset{p^{\text{DA}}_{i,t}, p^{\text{RT}}_{i,t,s}}{\text{Maximize}} \ \sum_t \left[p^{\text{DA}}_{i,t} \ \lambda^{\text{DA}}_{(n:i\in\Psi_n),t} \right. \\ \left. + \sum_s \phi_s \ p^{\text{RT}}_{i,t,s} \ \frac{\lambda^{\text{RT}}_{(n:i\in\Psi_n),t,s}}{\phi_s} \right] \qquad \text{(1ca)} \\ \text{subject to:} \\ p^{\text{DA}}_{i,t} \le W^{\text{DA}}_{i,t} \quad : \overline{\mu}_{i,t} \quad \forall t \qquad \text{(1cb)} \\ 0 \le \left(p^{\text{DA}}_{i,t} + p^{\text{RT}}_{i,t,s} \right) \le W^{\text{RT}}_{i,t,s} \quad : \underline{\rho}_{i,t,s}, \overline{\rho}_{i,t,s} \ \forall t, s \qquad \text{(1cc)} \\ p^{\text{DA}}_{i,t} \ge 0 \quad \forall t \end{array} \right\} \ \forall i \in \mathcal{W}. \qquad \text{(1cd)}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2017.2699687, IEEE Transactions on Power Systems

Objective function (1ca) maximizes the expected profit of wind generator $i \in \mathcal{W}$ in the DA and RT markets. In addition, constraints (1cb)-(1cd) enforce its production limits in DA and RT markets. Note that constraint (1cc) implicitly allows the excessive wind power to be spilled. Note also that the upper bound of (1cc), i.e., stochastic parameter $W^{\mathrm{RT}}_{i,t,s}$, represents the scenarios of potential RT wind power output.

The profit-maximization problem for each slow DR provider $d \in \mathcal{SDR}$ is given by (1d) below:

$$\left\{\begin{aligned} &\underset{d^{\mathrm{DA}\downarrow}_{d,k,t},d^{\mathrm{DA}\uparrow}_{d,k,t}}{\text{Maximize}} \quad \sum_{k,t}\left[d^{\mathrm{DA}\downarrow}_{d,k,t}\left(\lambda^{\mathrm{DA}}_{(n:d\in\Psi_n),t}-C^{\downarrow}_{d,k,t}\right)\right.\\ &\left.+\,d^{\mathrm{DA}\uparrow}_{d,k,t}\left(C^{\uparrow}_{d,k,t}-\lambda^{\mathrm{DA}}_{(n:d\in\Psi_n),t}\right)\right] \qquad \text{(1da)}\\ &\text{subject to:}\\ &d^{\mathrm{DA}\downarrow}_{d,k,t}\le D^{\downarrow}_{d,k,t} \quad :\mu^{\downarrow}_{d,k,t} \quad \forall k,t \qquad \text{(1db)}\\ &d^{\mathrm{DA}\uparrow}_{d,k,t}\le D^{\uparrow}_{d,k,t} \quad :\mu^{\uparrow}_{d,k,t} \quad \forall k,t \qquad \text{(1dc)}\\ &d^{\mathrm{DA}\downarrow}_{d,k,t}\ge 0;\; d^{\mathrm{DA}\uparrow}_{d,k,t}\ge 0 \quad \forall k,t \end{aligned}\right\} \forall d\in\mathcal{SDR}. \qquad \text{(1dd)}$$

Objective function (1da) maximizes the profit of slow DR provider $d \in \mathcal{SDR}$ in the DA market. Note that this DR resource does not provide balancing services in RT. Constraints (1db)-(1dd) bind its downward and upward DR quantities.

The profit-maximization problem for each fast DR provider $d \in \mathcal{FDR}$ is given by (1e) below:

$$\left\{\begin{aligned} &\underset{d^{\mathrm{DA}\downarrow}_{d,k,t},d^{\mathrm{DA}\uparrow}_{d,k,t},d^{\mathrm{RT}\downarrow}_{d,k,t,s},d^{\mathrm{RT}\uparrow}_{d,k,t,s}}{\text{Maximize}}\\ &\sum_{k,t}\left[d^{\mathrm{DA}\downarrow}_{d,k,t}\left(\lambda^{\mathrm{DA}}_{(n:d\in\Psi_n),t}-C^{\downarrow}_{d,k,t}\right)\right.\\ &\left.+\,d^{\mathrm{DA}\uparrow}_{d,k,t}\left(C^{\uparrow}_{d,k,t}-\lambda^{\mathrm{DA}}_{(n:d\in\Psi_n),t}\right)\right]\\ &+\sum_{k,t,s}\phi_s\left[d^{\mathrm{RT}\downarrow}_{d,k,t,s}\left(\frac{\lambda^{\mathrm{RT}}_{(n:d\in\Psi_n),t,s}}{\phi_s}-C^{\downarrow}_{d,k,t}\right)\right.\\ &\left.+\,d^{\mathrm{RT}\uparrow}_{d,k,t,s}\left(C^{\uparrow}_{d,k,t}-\frac{\lambda^{\mathrm{RT}}_{(n:d\in\Psi_n),t,s}}{\phi_s}\right)\right] \qquad \text{(1ea)}\\ &\text{subject to:}\\ &\text{(1db)}-\text{(1dd)} \qquad \text{(1eb)}\\ &0\le\left(d^{\mathrm{DA}\downarrow}_{d,k,t}+d^{\mathrm{RT}\downarrow}_{d,k,t,s}\right)\le D^{\downarrow}_{d,k,t}:\underline{\rho}^{\downarrow}_{d,k,t,s},\overline{\rho}^{\downarrow}_{d,k,t,s}\\ &\qquad\qquad\qquad\qquad\forall k,t,s \qquad \text{(1ec)}\\ &0\le\left(d^{\mathrm{DA}\uparrow}_{d,k,t}+d^{\mathrm{RT}\uparrow}_{d,k,t,s}\right)\le D^{\uparrow}_{d,k,t}:\underline{\rho}^{\uparrow}_{d,k,t,s},\overline{\rho}^{\uparrow}_{d,k,t,s}\\ &\qquad\qquad\qquad\qquad\forall k,t,s \end{aligned}\right\} \forall d\in\mathcal{FDR}. \qquad \text{(1ed)}$$

Objective function (1ea) maximizes the expected profit of fast DR provider $d \in \mathcal{FDR}$ in the DA and RT markets. Note that unlike the slow DR providers, each fast DR provider can participate in both DA and RT markets. Constraints (1eb) bind its upward and downward DR quantities in the DA market, while similar constraints in the RT market are enforced by (1ec) and (1ed).

The profit-maximization problem for each virtual bidder f is given by (1f) below:

$$\left\{\begin{aligned} &\underset{v^{\mathrm{DA}}_{f,t},v^{\mathrm{RT}}_{f,t}}{\text{Maximize}} \quad \sum_{t}\left[v^{\mathrm{DA}}_{f,t}\ \lambda^{\mathrm{DA}}_{(n:f\in\Psi_n),t}\right.\\ &\left.+\sum_{s}\phi_s\ v^{\mathrm{RT}}_{f,t}\ \frac{\lambda^{\mathrm{RT}}_{(n:f\in\Psi_n),t,s}}{\phi_s}\right] \qquad \text{(1fa)}\\ &\text{subject to:} \quad v^{\mathrm{DA}}_{f,t}+v^{\mathrm{RT}}_{f,t}=0 \quad :\rho_{f,t} \quad \forall t \end{aligned}\right\} \forall f. \qquad \text{(1fb)}$$

Objective function (1fa) maximizes the expected profit of virtual bidder f participating in the DA and RT markets. Note that the trading quantities in DA and RT markets, i.e., $v^{\mathrm{DA}}_{f,t}$ and $v^{\mathrm{RT}}_{f,t}$, are both scenario-independent since virtual bidders own no physical assets and so are obliged to zero out their financial position. In other words, as constrained by (1fb), each virtual bidder buys/sells a quantity in the DA market and then sells/buys back that quantity in the RT market. Note also that (1f) implicitly imposes the equality of DA and expected RT prices at bus n within the equilibrium problem. It is straightforward to yield this price equality condition through the Karush-Kuhn-Tucker (KKT) conditions of (1f).

The profit-maximization problem for the ISO is given by (1g). Note that the following formulation is based on the $B-\theta$ formulation of the linearized DC load flow, and is equivalent to the power transfer distribution factor (PTDF)-based grid operator problem in [36]:

$$\underset{a^{\mathrm{DA}}_{n,m,t},\theta^{\mathrm{DA}}_{n,t},a^{\mathrm{RT}}_{n,m,t,s},\theta^{\mathrm{RT}}_{n,t,s}}{\text{Maximize}}$$

$$\sum_{n,m\in\Omega_n,t}\left[a^{\mathrm{DA}}_{m,n,t}\ \lambda^{\mathrm{DA}}_{n,t}+\sum_{s}\phi_s\left(a^{\mathrm{RT}}_{m,n,t,s}-a^{\mathrm{DA}}_{m,n,t}\right)\frac{\lambda^{\mathrm{RT}}_{n,t,s}}{\phi_s}\right] \qquad \text{(1ga)}$$

subject to:

$$a^{\mathrm{DA}}_{n,m,t}=B_{n,m}\left(\theta^{\mathrm{DA}}_{n,t}-\theta^{\mathrm{DA}}_{m,t}\right) \quad :\mu_{n,m,t}\ \forall n,m\in\Omega_n,t \qquad \text{(1gb)}$$

$$a^{\mathrm{RT}}_{n,m,t,s}=B_{n,m}\left(\theta^{\mathrm{RT}}_{n,t,s}-\theta^{\mathrm{RT}}_{m,t,s}\right) \quad :\rho_{n,m,t,s} \quad \forall n,m\in\Omega_n,t,s \qquad \text{(1gc)}$$

$$a^{\mathrm{DA}}_{n,m,t}\le F_{n,m} \quad :\mu^{\mathrm{flow}}_{n,m,t} \quad \forall n,m\in\Omega_n,t \qquad \text{(1gd)}$$

$$a^{\mathrm{RT}}_{n,m,t,s}\le F_{n,m} \quad :\rho^{\mathrm{flow}}_{n,m,t,s} \quad \forall n,m\in\Omega_n,t,s \qquad \text{(1ge)}$$

$$\theta^{\mathrm{DA}}_{(n=1),t}=0 \quad :\mu^{1}_{t} \quad \forall t \qquad \text{(1gf)}$$

$$\theta^{\mathrm{RT}}_{(n=1),t,s}=0 \quad :\rho^{1}_{t,s} \quad \forall t,s. \qquad \text{(1gg)}$$

Objective function (1ga) maximizes the expected profit that the ISO gains by transferring energy throughout the network in DA and RT markets. Constraints (1gb) and (1gc) define the power flow across each transmission line in DA and RT markets, respectively. Constraints (1gd) and (1ge) enforce the line limits in the two markets. Finally, constraints (1gf) and (1gg) introduce node $n=1$ as the voltage angle reference node. Note that the physical load flow and flow capacity constraints are enforced in both DA and RT markets, although

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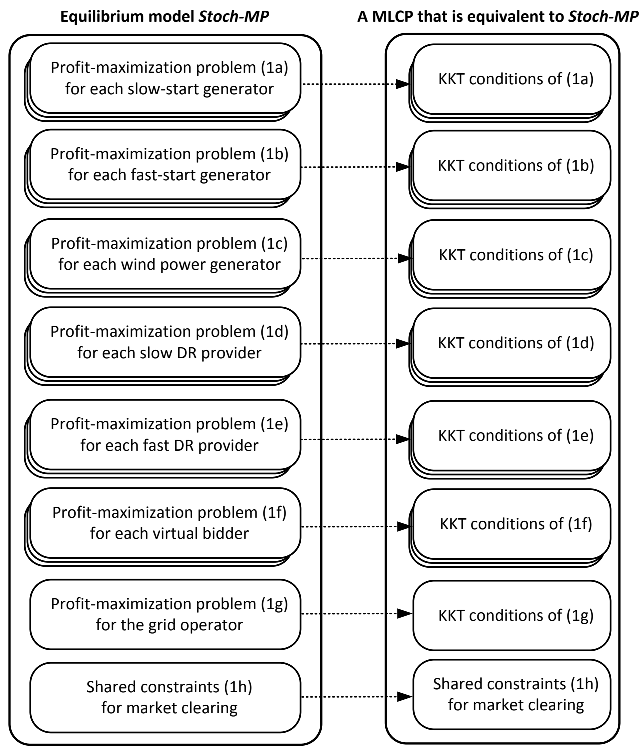

Fig. 8. Model $Stoch\text{-}MP$ recast as a mixed linear complementarity problem (MLCP).

the DA constraints could be omitted without changing the equilibrium profits and prices.

Finally, the market-clearing constraints in the DA and RT markets are respectively given by (1ha) and (1hb) below:

$$\sum_{d\in\Psi_n,k}\left(L_{d,t}+d^{\mathrm{DA}\uparrow}_{d,k,t}-d^{\mathrm{DA}\downarrow}_{d,k,t}\right)+\sum_{m\in\Omega_n}a^{\mathrm{DA}}_{n,m,t}-\sum_{i\in\Psi_n}p^{\mathrm{DA}}_{i,t} \\ -\sum_{f\in\Psi_n}v^{\mathrm{DA}}_{f,t}=0 \quad :\lambda^{\mathrm{DA}}_{n,t}\quad\forall n,t \tag{1ha}$$

$$\sum_{d\in\Psi_n,k}\left(d^{\mathrm{RT}\uparrow}_{d,k,t,s}-d^{\mathrm{RT}\downarrow}_{d,k,t,s}\right)+\sum_{m\in\Omega_n}\left(a^{\mathrm{RT}}_{n,m,t,s}-a^{\mathrm{DA}}_{n,m,t}\right) \\ -\sum_{i\in\Psi_n}p^{\mathrm{RT}}_{i,t,s}-\sum_{f\in\Psi_n}v^{\mathrm{RT}}_{f,t}=0 \quad :\lambda^{\mathrm{RT}}_{n,t,s}\quad\forall n,t,s. \tag{1hb}$$

The DA balancing constraint (1ha) implies that at each node and in each hour, the load based on DR schedules is equal to dispatchable and wind generation plus injection of arbitragers into the grid and net power inflows from other nodes. The RT production forecast errors are accommodated in (1hb) by generator redispatch, fast generator commitment, and fast DR schedules. Note that the dual variables of (1ha) and (1hb) provide day-ahead and probability-weighted real-time LMPs, respectively.

The solution of $Stoch\text{-}MP$, i.e., the multi-player stochastic two-settlement Nash-Bertrand equilibrium point, can be obtained by simultaneously solving the KKT conditions of all players together with market clearing [36], as illustrated in Fig. 8. The resulting model is a mixed linear complementarity problem (MLCP) that can be solved using PATH or other complementarity problem solvers. This is a Nash-Bertrand equilibrium because each player believes it cannot affect prices by its actions; this model could be generalized to a closed loop Nash-Cournot model if some or all players recognize how quantities supplied/purchased affect prices through the demand function. The equilibrium conditions of $Stoch\text{-}MP$ (i.e., the KKT and market-clearing conditions) are described in Appendix of [48]. A general introduction to equilibrium and complementarity models can be found in [49].

IV. Conclusions

In the first part of this two-paper series, four different models for clearing the two-settlement markets (day-ahead and real-time) are proposed. These include (i) a multi-player stochastic equilibrium in which all players have the same beliefs about the distribution of RT prices, (ii) co-optimization of all schedules by the ISO using stochastic UC, (iii) a sequential equilibrium in which the operator first selects DA schedules considering a deterministic wind forecast, and then rebalances the market in RT against the actual wind realization, and (iv) an extension of the sequential equilibrium with some self-scheduling slow-start generators. The assumptions of each are summarized, and the players' individual profit maximizing models that make up the first model are presented.

These four models provide a framework to quantify the economic value of flexible resources. In addition, the models can address whether grid operators must use stochastic UC models to achieve the optimal solution, or whether virtual bidders along with some self-scheduling generators could obtain it. The companion paper [48] provides the formulation of $Stoch\text{-}Opt$, Seq and $Seq\text{-}SS$ as well as results from a simple illustrative example and a large-scale case study that quantify the economic value of different types of flexible resources. The applications also confirm that, indeed, the errors of deterministic DA scheduling can be corrected by virtual bidders and a few self-scheduled generators.

Acknowledgement

We thank Venkat Prava for research assistance, and Judy Cardell and C. Lindsay Anderson for useful discussions. We also thank the three anonymous reviewers for their helpful comments.

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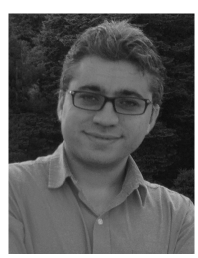

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